CRITICAL CURRENT DENSITY IN RAIL ACCELERATORS WITH A PLASMA PISTON

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In recent years researchers working in the field of pulsed power production and high-velocity projectiles have devoted a great deal of attention to the study of the potential possibilities of the electrodynamic method for accelerating solid bodies in rail accelerators with a plasma piston and a dielectric body. International conferences [1, 2] and a national symposium in the USA [3] were devoted to these questions. It is expected that the use of electromagnetic forces for accelerating macroscopic particles will make it possible to achieve velocities significantly exceeding the values achieved by other methods. The possibility of constructing devices for accelerating particles with a mass of the order of one gram up to different velocities was studied and designs were discussed in a number of works: 12 km/sec [4], 15 km/sec [5, 6], 20 km/sec [7], 25 km/sec [5, 8], etc.

The authors of these works believe that the main advantage of acceleration of dielectric solid bodies with a plasma piston lies in the possibility of removing the thermal limitation, arising due to the electric current flowing in the circuit, on the velocity of metallic particles. Thus in [7] Hawke and Scudder, evaluating the maximum velocity up to which cubic copper particles can be accelerated in a rail accelerator (≈9 km/sec), conclude that in order to achieve higher velocities acceleration of dielectric solids with a plasma piston must be employed.

In studying the possibilities of the rail accelerator method for accelerating solids one of the most important problems is to determine the critical current density per unit width of the electrode, above which factors limiting the operation of the accelerator start to operate: destruction of the projectile, destruction of the accelerator channel, melting and vaporization of the surface of the electrodes, etc. A detailed analysis of this problem has not been published. Some estimates of the critical values of I_0/b (I_0 is the current flowing in the circuit and b is the width of the electrodes) are given in [7], where the estimate $I_0/b \le 81$ MA/m was obtained based on the strength of the existing dielectrics fabricated based on rubber and graphite. The critical current density, above which copper electrodes melt owing to Joule heating, is estimated to be 43 MA/m with a stepped growth of the magnetic field.

In this paper we shall study the critical current density at which the temperature of the surface of the electrodes reaches the melting temperature as a result of the heating of the electrodes by the current flowing in the circuit and the plasma piston accelerating the dielectric body. The dependences of the critical current density I_0/b on the basic physical properties of the electrode material and the plasma and rail accelerator parameters are determined.

1. A diagram of the rail accelerator with a plasma piston is shown in Fig. 1; 1 is the source of electric power, 2 is the accelerated dielectric body, 3 is the plasma piston, 4 are the electrodes, ℓ is the length of the plasmoid, d is the distance between the electrodes, and v is the velocity of the body and plasmoid. The problem is to determine the maximum admissible current flowing in the circuit, for which the surface of the electrodes will reach the melting point. In the scheme under study the temperature of the electrodes (heat energy) increases under the action of the internal (Joule heating) and external (plasma) heat sources. The most complete solution of this problem obviously can be obtained only by numerical methods. We shall make a number of assumptions in order to obtain analytical estimates. We shall assume that the dimensions of the plasmoid remain unchanged during the motion (solid-body model) and $b = d$, $\ell = kb$ ($k = \text{const}$). The transient processes associated with establishing the current in the circuit and starting the body in motion are ignored. We

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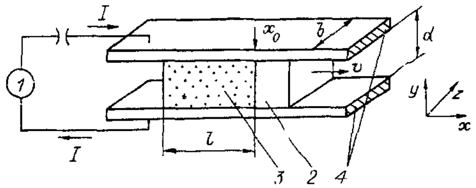

Fig. 1

single out the point x_0 on the electrode. Assume that at time t = 0 the plasma piston approaches the point x_0. We denote by v_0 and I_0 the velocity of the plasma and current in the circuit at this moment. We assume that the current in the circuit remains constant. Under these assumptions we shall determine the critical current density I_0/b at which within the time of transit of the plasmoid past the point x_0 the temperature at this point reaches the melting point of the electrodes T_m.

It is obvious that with this formulation of the problem the values obtained for I_0/b will be too high, since the Joule heating at the point x_0 will continue after the plasma passes this point. We shall determine the transit time of the plasma past the point x_0 from the equation

$$\int_0^{t_0} v dt = l. \quad (1.1)$$

Based on the adopted assumptions, neglecting friction and the counterpressure, the velocity of the body is given by

$$v = v_0 + \lambda_r I_0^2 t/2m. \quad (1.2)$$

Here m is the mass of the accelerated body and λ_r is the inductance of the rail accelerator per unit length. Substituting (1.2) into (1.1) and carrying out the calculations we find that the transit time of the plasmoid past the point x_0 is given by

$$t_0 = \frac{2l}{v_0}\left(\frac{\sqrt{1+A}-1}{A}\right) \quad (A = \lambda_r I_0^2 l/(m v_0^2)). \quad (1.3)$$

As $A \to 0$, $t_0 \to \ell/v_0$; for $A \gg 1$, $t_0 \approx 2kb/(v_0\sqrt{A})$.

We shall determine the change in the temperature of the electrode T at the point x_0 from the solution of the one-dimensional heat-conduction equation

$$\rho c \partial T/\partial t = \lambda \partial^2 T/\partial y^2 + j^2/\sigma \quad (1.4)$$

with a fixed starting temperature distribution and with a boundary condition of the type for a boundary-value problem of the second kind:

$$T(y; 0) = 0, \ \partial T(0; t)/\partial y = -F/\lambda, \quad (1.5)$$

where ρ, c, λ, and σ are the density, heat capacity, thermal conductivity, and electric conductivity of the electrode material; j is the current density; and, F is the heat flux from the plasma into the conductor. We assumed that ρ, c, and λ remain constant as the plasma passes the point x_0.

Using the standard procedure, the solution of the boundary-value problem (1.4) can be represented as a sum of solutions of two boundary-value problems:

$$\begin{gathered}\rho c \partial T_1/\partial t = \lambda \partial^2 T_1/\partial y^2, \ T_1(y; 0) = 0, \\ \partial T_1(0; t)/\partial y = -F/\lambda, \ 0 \leqslant t \leqslant t_0;\end{gathered} \quad (1.6)$$

$$\begin{gathered}\rho c \partial T_2/\partial t = j^2/\sigma, \ T_2(y; 0) = 0, \\ \partial T_2(0; t)/\partial y = 0, \ 0 \leqslant t \leqslant t_0.\end{gathered} \quad (1.7)$$

The equations (1.6) and (1.7) describe, respectively, the change in the temperature owing to the external heat source (effect of the plasma piston) and the internal heat source (Joule heating).

TABLE 1

r	$\varphi_1(r)$
0	1,3
1	0,88
2	0,75
4	1,07
8	0,81
10	0,74

TABLE 2

n	$\varphi_2(n)$
1	0,78
2	0,64
4	0,56
10	0,52

2. We shall study the solution of the boundary-value problem (1.6). To determine T_1 it is necessary to know the heat flux as a function of time or length of the plasma piston as the plasma passes the point x_0. We shall represent the change in F in the form of a power-law function

$$F = F_0(t/t_0)^{r/2} \tag{2.1}$$

(r can equal −1.0 or some positive integer). The dependence F(r) makes it possible to evaluate the effect of the heat flux curves on the change in T_1. An expression for the temperature on the surface of an electrode can be obtained from the solution of Eq. (1.6) with the initial and boundary conditions (1.5) and (2.1) [9]:

$$T_1(0;\ t) = \frac{F_0}{(\lambda\rho c)^{1/2}} \frac{\Gamma\left(\frac{r}{2}+1\right)}{\Gamma\left(\frac{r}{2}+\frac{3}{2}\right)} \left(\frac{t}{t_0}\right)^{r/2} t^{1/2},$$

where $\Gamma(r/2+1)$ and $\Gamma(r/2+3/2)$ are gamma functions. At t = t_0 $T_1(0;\ t_0)$ depends on r only through the ratio of the indicated γ functions, and in addition the change in $\varphi_1(r) = \Gamma(r/2+1)/\Gamma(r/2+3/2)$ for a wide range of values of r is very insignificant (Table 1). For this reason, in what follows we shall assume that

$$F = F_0 = \text{const.} \tag{2.2}$$

In this case

$$T_1(0;\ t_0) = F_0(4t_0/(\pi\lambda\rho c))^{1/2}. \tag{2.3}$$

We shall determine the value of F_0 from the energy balance in the plasmoid

$$I_0^2 R_p = d\varepsilon_p/dt + \int F dS. \tag{2.4}$$

Here R_p and ϵ_p are the resistance and internal energy of the plasma; S is the surface area of the plasmoid: $S = 4b^2k + 2b^2$. Setting $d\varepsilon_p/dt = 0$, we rewrite the expression (2.4), taking into account (2.2), in the form $I_0^2 R_p = 2F_0 b^2(2k+1)$, whence

$$F_0 = I_0^2 R_p/(2b^2(2k+1)). \tag{2.5}$$

Substituting (2.5) into (2.3), we obtain the dependence of the change in the temperature of the electrode surface heated by the moving plasma:

$$T_1(0;\ t_0) = \frac{I_0^2}{b^2}\frac{R_p}{2k+1}\sqrt{\frac{t_0}{\pi\lambda\rho c}}. \tag{2.6}$$

3. We shall study the solution of the boundary-value problem (1.7) for two cases of variation of σ.

A. $\sigma = \sigma_0$ = const. Using the Maxwell's equation $j = \partial H_z/\partial y$ we represent the change in the electrode temperature due to the current flowing in the circuit as

$$T_2(y;\ t) = 1/(\sigma_0\rho c)\int_0^t [\partial H_z/\partial y]^2\, dt. \tag{3.1}$$

TABLE 3

Element	λ, W/(m·deg)	ρ, 10^{-3} kg/m³	c, J/(kg·deg)	χ, 10^{5} m²/sec	T_m, °C	$\lambda\rho c T_m$, 10^{14} J·W/m⁴	I_0/b, 10^7 A/m
Cu	399	8,96	390	11,4	1083	16,31	1,05
Hf	22	13,36	140	1,17	2130	1,87	0,62
Fe	75,4	7,87	445	2,15	1539	6,25	0,84
Al	218	2,7	903	8,94	658,7	2,29	0,65
W	180 *	19,2	134	7,0	3395	53,77	1,45
Mo	151,7 **	10,2	253	5,88	2622	26,91	1,22
Ti	23,2	4,54	550	0,93	1668	1,61	0,60
Cr	67	7,19	452	2,06	1890	7,77	0,88

*At T = 0.
**At T = 100°C.

The derivative $\partial H_z/\partial y$ must be determined from the equation of diffusion of the magnetic field into the conductor at the point x_0

$$\partial^2 H_z/\partial y^2 = \sigma_0\mu_0 \partial H_z/\partial t. \tag{3.2}$$

For the initial condition we choose $H_z(y;\, 0) = 0$, and for the boundary condition we take the distribution of the magnetic field in the plasma up to the moment the plasmoid approaches the point x_0. Since it is unknown, we shall employ the boundary condition in the form of a power-law function

$$H_z = H_0(t/t_0)^{n/2},\ 0 \leqslant t \leqslant t_0, \tag{3.3}$$

where $H_0 = I_0/b$ and n is a positive integer. By varying the index n it is possible to determine the effect of the profile of the magnetic field in the plasma on the electrode temperature as the plasma passes the point x_0.

The solution of the diffusion equation (3.2) with the boundary condition (3.3) has the form [9]

$$H_z(y;\, t) = H_0\Gamma(n/2+1)(4t/t_0)^{n/2}\left\{i^n \operatorname{erfc}\frac{y\sqrt{\sigma_0\mu_0}}{2\sqrt{t}}\right\}. \tag{3.4}$$

Substituting $\partial H_z(y;\, t)/\partial y$, determined from (3.4), into (3.1) we find formulas for evaluating the temperature T_0 on the surface of the electrodes [10]:

$$T_2(0;\, t) = \frac{\mu_0}{\rho c}\frac{I_0^2}{b^2}\left(\frac{t}{t_0}\right)^n \frac{1}{n}\left[\frac{\Gamma(n/2+1)}{\Gamma(n/2+1/2)}\right]^2,$$

and at $t = t_0$

$$T_2(0;\, t_0) = \mu_0 I_0^2 \varphi_2(n)/(\rho c b^2),\ \ \varphi_2(n) = \left[\frac{\Gamma(n/2+1)}{\Gamma(n/2+1/2)}\right]^2\Big/n. \tag{3.5}$$

The function $\varphi_2(n)$ is virtually independent of n for n from 1 to 10 (Table 2). The ratio

$$T_2(0;\, t_0)/T_1(0;\, t_0) = (1+2k)\varphi_2(n)\mu_0\sqrt{\pi\chi}/R_p\sqrt{t_0} \tag{3.6}$$

($\chi = \lambda/\rho c$ is the thermal diffusivity). The combination (2.6) and (3.5) permits determining the critical current density I_0/b at which the surface reaches the melting point within the transit time of the plasma past the point x_0:

$$\frac{I_0}{b} = \left[\frac{(1+2k)^2\pi\lambda\rho c T_m^2}{R_p^2 t_0}\right]^{1/4}\frac{1}{(1+T_2/T_1)^{1/2}}\ (T_m = T_2 + T_1). \tag{3.7}$$

The values of χ and $\lambda\rho c T_m^2$ at 20°C are given in Table 3 for a number of metals of interest for use as electrodes [11].

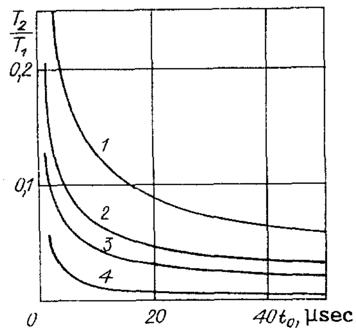

Fig. 2

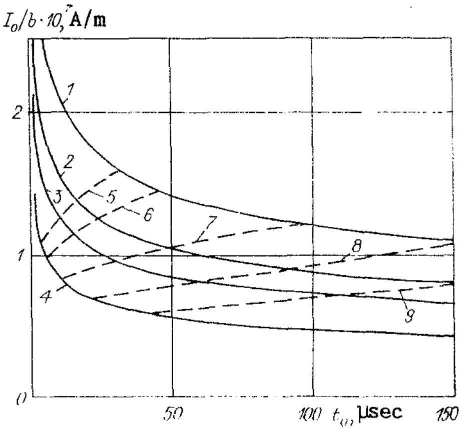

Fig. 3

B. $\sigma = \sigma_0/(1 + \beta\rho c T_2)$ (β is a thermal coefficient). The formulas (3.5)-(3.7), obtained for $\sigma = \sigma_0 = \text{const}$, do not change significantly, when the electric conductivity depends on the heat absorbed by the electrode material on heating.

In this case, under the boundary condition (3.3) the temperature on the surface of the conductor [12] $T_2(0;\ t) \simeq \frac{1}{2}\frac{\mu_0 I_0^2}{\rho c\, b^2}\left(\frac{t}{t_0}\right)^n (1 + 1/2n)$, which for the range of n studied agrees with (3.5) with good accuracy.

Thus the expression (3.7) together with (1.3) and (3.6) permits analyzing the dependence of the critical current density I_0/b on the physical properties of the electrode material (λ, ρ, c, T_m), the plasma parameters (R_p, $\varphi_2(n)$, k, v_0), and the parameters of the rail accelerator (λ_r, m, b).

4. The dependence of $T_2(0;\ t_0)/T_1(0;\ t_0)$ on t_0 for copper electrodes with $\chi_{Cu} = 11.4\cdot 10^{-5}$ m^2/sec, $R_p = 10^{-3}\Omega$, and $\varphi_2(n) = 0.78$ are presented in Fig. 2 for k = 10, 5, 3 and 1 (curves 1-4). The value of the resistance of the plasma R_p is taken from [13], and the value of χ_{Cu} is taken from [11] for 20°C. It is obvious from the graph that for $t_0 \gtrsim 2$—5 μsec $T_2/T_1 \ll 1$ and can be neglected in the expression (3.7). For constant values R_p, t_0, k and $\varphi_2(n)$ the ratio T_2/T_1 for an arbitrary material i can be represented in the form $(T_2(0;\ t_0)/T_1(0;\ t_0))_i = (T_2(0;\ t_0)/T_1(0;\ t_0))_{Cu}(\chi_i/\chi_{Cu})^{1/2}$. For the metals shown in Table 3, $\chi_i < \chi_{Cu}$ and $(T_2(0;\ t_0)/T_1(0;\ t_0))_i < (T_2(0;\ t_0)/T_1(0;\ t_0))_{Cu}$.

The dependence of I_0/b on t_0 from (3.7) for copper electrodes with $R_p = 10^{-3}\Omega$, $\varphi_2(n) = 0.78$, is shown in Fig. 3 for k = 10, 5, 3, and 1 (lines 1-4). The values of λ, ρ, and c at 20°C were used [11]. The intersection of these curves with the curve of I_0/b versus $t_0 = f(m, \lambda_r, b, v_0, k)$ permits determining from (1.3) concrete values of I_0/b and t_0, for which the temperature of the electrodes at the point x_0 reaches T_m within the transit time of the plasma past this point. The values of I_0/b for acceleration of bodies with a mass of 10^{-3} kg and b = 10^{-2} m, $\lambda_r = 0.25\times 10^{-6}$. H/m are shown in Fig. 3 by the lines 5-9, corresponding to v_0 = 3, 2, 1, and 0.5 km/sec. It is obvious that for copper electrodes and the adopted parameters of the plasma and accelerator the critical current density must not exceed (1-1.5)$\cdot 10^7$ A/m.

We shall study the analytic dependence of I_0/b on the physical properties of the electrode material, and the parameters of the plasma and rail accelerator. Substituting into (3.7) the value of t_0 as A → 0 and for A >> 1 we find the following asymptotic expressions for $T_2/T_1 \ll 1$:

$$\frac{I_0}{b} = \begin{cases} (\alpha v_0/(kb))^{1/4} & \text{at } A \to 0, \\ (\alpha^2 \lambda_r b/(4mk))^{1/6} & \text{at } A \gg 1 \end{cases}$$

$$(\alpha = \pi (2k+1)^2 \lambda\rho c T_m^2 / R_p^2).$$

For small values of A the critical current density is independent of λ_r and m, while for A >> 1 it is independent of v_0. Substituting (1.3) into (3.7) and assuming that $T_2/T_1 \ll 1$ we obtain the "incomplete" cubic equation

$$y^3 + py + q = 0,\ y = (I_0/b)^2,\ p = -\alpha v_0/(kb),\quad q = -\alpha^2 \lambda_r b/(4mk). \tag{4.1}$$

The real roots of Eq. (4.1) can be easily found. We confine the analysis to cases when the discriminant $D = (p/3)^3 + (q/2)^2 = 0$ and D < 0. For D = 0 (4.1) has one real root [14], satisfying the conditions of the problem at hand:

$$I_0/b = (\alpha^2\lambda_r b/k)^{1/6}, \tag{4.2}$$

while the parameters are related by the relation

$$64m^2v_0^3/(27k\alpha b^5\lambda_r^2) = 1. \tag{4.3}$$

Substituting in (4.2) the value of α and taking into account (4.3), we find

$$\frac{I_0}{b} = \left\{\frac{8\pi}{27}\frac{m}{\lambda_r b^4}\frac{v_0^3(2k+1)^2}{k^2R_p^2}\lambda\rho c T_m^2\right\}^{1/6}. \tag{4.4}$$

The cofactors in (4.4) consist of the parameters characterizing the accelerator, plasma, and electrode material, respectively. For typical values of the accelerator parameters usually D < 0. In this case (4.1) also has one real root [14]:

$$y = 2\sqrt[3]{a}\cos(\varphi/3)\ \left(a = \sqrt{-p^3/27},\ \cos\varphi = -q/2a\right). \tag{4.5}$$

It is easy to verify that for a wide range of system parameters (in particular, for parameters used to construct Fig. 3) $-q/2a \ll 1$. Expanding cos φ in a series and substituting the value of φ in (4.5), we obtain $y = (\alpha v_0/(kb))^{1/2} + \alpha\lambda_r b^2/(8mv_0)$, whence

$$I_0/b = [(\alpha v_0/kb)^{1/2} + \lambda_r\alpha b^2/(8mv_0)]^{1/2}. \tag{4.6}$$

Substituting the value of α into (4.6), we find

$$\frac{I_0}{b} = \left[\left(\frac{\pi(2k+1)^2 v_0\lambda\rho c T_m^2}{R_p^2 kb}\right)^{1/2} + \frac{\pi(2k+1)^2\lambda_r b^2}{8mv_0R_p^2}\lambda\rho c T_m^2\right]^{1/2}. \tag{4.7}$$

The values of I_0/b, calculated using the formula (4.7), agree with high accuracy with the values of I_0/b obtained by solving the system (1.3) and (3.7) on a computer (see Fig. 3).

The values of I_0/b, calculated from the formula (4.7), are presented in Table 3 for a number of metals with $m = 10^{-3}$ kg, $\lambda_r = 0.25\cdot10^{-6}$ H/m, $b = 10^{-2}$ m, $v_0 = 10^3$ m/sec, $k = 5$, $R_p = 10^{-3}$. It follows from the table that the critical current density cannot be significantly improved by replacing copper electrodes by electrodes made of the materials studied.

The foregoing analysis shows that the plasma piston makes the main contribution to the increase in the temperature of the electrodes in a rail accelerator accelerating solid bodies with a plasma piston over a wide range of parameters; this significantly affects I_0/b. The estimates obtained for the critical current density are significantly lower than the estimated values of I_0/b obtained in [7]. Since the accelerating force in a rail accelerator is proportional to $(I_0/b)^2$, small values of the critical current density in the accelerator with the plasma piston make it necessary to employ incredibly long rail accelerators; this casts doubt on the desirability of using a plasma piston to accelerate solid bodies.

We shall evaluate the required length of an accelerator with copper electrodes for obtaining a particle velocity of $v = 10^4$ m/sec for $m = 10^{-3}$ kg, $v_0 = 10^3$ m/sec, $b = 10^{-2}$ m, $\lambda_r = 2.5\cdot10^{-7}$ H/m, and $I_0/b = 10^7$ A/m (see Fig. 3 and Table 3). Substituting the expression (1.3) into (1.2), replacing t_0 by t, and ℓ by the length of the accelerator X, we have $X = [(v^2 - v_0^2)m/(\lambda_r b^2)]b^2/I_0^2$, whence it follows that for the indicated parameters $X \approx 40$ m For $v = 15\cdot10^3$ m/sec and the same starting parameters $x \approx 90$ m; for $v = 5\cdot10^3$ m/sec, $X \approx 10$ m. These estimates show that if the current density does not exceed the critical value the length of the accelerator becomes unacceptable from the standpoint of practical realization. To shorten the rail accelerator the current flowing in the circuit must be reduced. Then, however, the current density per unit channel width exceeds the critical current density, the temperature of the electrodes will exceed the melting temperature (possibly also the boiling temperature), erosion of the electrodes will start, and the picture of the physical processes occurring in the accelerator will change significantly.

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